Respostas da Lista de Exercícios XV

1. (a) $m\gamma v = eEt$.
   (b) $v = \beta c = \dfrac{eEct}{\sqrt{(eEt)^2 + (mc)^2}}$

2. Defina $\eta^\mu := \frac{1}{K} K^\mu$, em que $K^\mu = (\omega/c, k_1, k_2, k_3)$. Note que $\eta^\mu F_{\mu\nu} = 0 \leftrightarrow K^\mu F_{\mu\nu} = 0$. Para $\nu = 0$:

$$k_1 E_1 + k_2 E_2 + k_3 E_3 = \vec{k} \cdot \vec{E} = 0,$$

   idem para mostrar $\vec{k} \cdot \vec{B} = 0$.

3. (a) Em S, os campos são:

$$\vec{E} = 0,$$

$$\vec{B} = \begin{cases} \dfrac{\mu_0 i r}{2\pi r_0^2} \hat{e}_\phi, \ (r \leq r_0) \\ \dfrac{\mu_0 i}{2\pi r} \hat{e}_\phi, \ (r > r_0), \end{cases}$$

   em que r_0 é o raio do cabo condutor. Os campos em S' são obtidos via a transformação de Lorentz, isto é:

$$E'_{||} = 0 = E_{||}, \qquad B'_{||} = B_{||} = 0,$$

$$E' = E_\perp = \gamma(E_\perp + v \times B_\perp) = -\gamma v B \hat{e}_r,$$

$$B' = B_\perp = \gamma(B_\perp - \frac{v \times B_\perp}{c^2}) = -\gamma B \hat{e}_\phi,$$

   (b) $\rho' = -\frac{vi\gamma}{\pi r_0^2 c^2}$.
   (c) $v'_e = -\frac{v+u}{1+\frac{uv}{c^2}} \hat{e}_x$, $v'_i = -v\hat{e}_x$.
   (d)

4. (a) $\lambda' = \gamma \lambda_0 \frac{u^2}{c^2}$.
   (b) $F_E = 0$, $F_B = quB = \frac{q\mu_0 \lambda_0 u^2}{2\pi r}$.
   (c) $F'_E = \frac{q\lambda'}{2\pi\epsilon_0 r}$, $F'_B = 0$.
   (d) $F'_E = F_B$.

5.

6. (a) $E(x,y,z,t) = E_0 \cos(kx - \omega t)\hat{y}$, $B(x,y,z,t) = \frac{E_0}{c}\cos(kx - \omega t)\hat{z}$.
   (b) $E'_x = E'_z = 0$, $E'_y = \gamma E_0 \cos(kx - \omega t)(1 - \frac{v}{c})$
   (c) $B'_x = B'_y = 0$, $B'_z = \gamma \frac{E_0}{c}\cos(kx - \omega t)(1 - \frac{v}{c})$.
   Note que $E'(x', y', z', t') = E'_0 \cos(k'x' - \omega' t')\hat{y}$ e $B(x', y', z', t') = \frac{E'_0}{c}\cos(k'x' - \omega' t')\hat{z}$, onde:

$$E'_0 = \alpha E_0, \quad k' = \alpha k, \quad \omega' = \alpha\omega,$$

   $\alpha = \sqrt{\frac{1-v/c}{1+v/c}}$
   (d) $\omega' = \alpha\omega$, $\lambda' = \frac{\lambda}{\alpha}$ e $v' = \omega'/k' = c$.

7. $E = 4mc^2$.

8. (a)
   (b)
   (c) $4m_0$.

9. $\vec{\nabla} \cdot \vec{B} = 0$
   e
   $\vec{\nabla} \times \vec{E} + \dfrac{\partial \vec{B}}{\partial t} = 0$

10. $(1+\beta^2)(1-\beta^2)^{-1} M_0 c^2$.

11. (a) $|\vec{p}_1| = \sqrt{\dfrac{m^4 + m_1^4 + m_2^4 - 2m^2 m_1^2 - 2m^2 m_2^2 - 2m_1^2 m_2^2}{4m^2}}$.
    (b) $E_1 = \dfrac{m^2 + m_1^2 - m_2^2}{2m}$, $E_2 = \dfrac{m^2 - m_1^2 + m_2^2}{2m}$.
    (c) $|\vec{p}_1| = \dfrac{\mu^2|\vec{p}|\cos\theta + \sqrt{\mu^4|\vec{p}|^2\cos^2\theta - (m_1^2E^2 - \mu^4)(E^2 - |\vec{p}|^2\cos^2\theta}}{E^2 - |\vec{p}|^2\cos^2\theta}$.