Introduction to Particle Physics
List 1

1 What is a quantum field theory?

2 Interactions in Quantum Field Theory

1. Derive Eq.(2.29) using the Dirac equation.

2. Consider the following interaction Lagrangian:

$$\mathcal{L}_{int} = y\phi\bar{\psi}\psi + h.c. \tag{1}$$

   where ϕ is a real scalar and ψ a Dirac Fermion. They have masses m_ϕ and m_ψ, respectively.

   - Compute the cross section $\psi\bar{\psi} \to \psi\bar{\psi}$;
   - Compute the decay width $\phi \to \psi\bar{\psi}$.

3. Repeat the computations of the previous exercise considering the following Lagrangian between a massive vector V_μ and a Dirac fermion ψ:

$$\mathcal{L}_{int} = yV_\mu\bar{\psi}\gamma^\mu\psi + h.c. \tag{2}$$

3 Quantum Electrodynamics

1. Derive Eqs.(3.48)-(3.49) using `Package X`.

2. Derive Eq.(3.71).

3. Do explicitly the 1-loop computation in Eq.(3.80) using `Package-X` (or by hand, if you prefer).

4. Derive the results presented in Table 3.1.

5. Show that the combination between brackets in Eq. (4.12) is the symmetric combination.
6. Show that the singlet S defined in Eq. (4.22) does not transform.
7. Show that the singlet combination appearing in the $\mathbf{3} \times \mathbf{3} \times \mathbf{3}$ product is
$$\epsilon^{ijk} u_i v_j w_k \ .$$
8. Compute explicitly the expression of the $\mathbf{8}_{M_A}$ and $\mathbf{8}_{M_S}$ representations appearing in Eq. (4.30) in terms of the components of u, v and w.

4 The quark model and $SU(3)$

1. Show that the matrix Π (Eq. (4.35)) has exactly the form shown in terms of the mesons defined in Eq. (4.34).

5 Quantum Chromo Dynamics (QCD)

1. Check explicitly that theJacobi identity of Eq. (5.8) allows to define matrices that satisfy the commutation relation shown in Eq. (5.9).
2. Prove that the identity $(\partial_\mu U)U^\dagger = -U(\partial_\mu U^\dagger)$ is true, where U is a non-abelian (local) transformation.

6 Confinement, the emergence of mesons and spontaneous symmetry breaking

1. Show explicitly that the quark mass term in the QCD Lagrangian is not invariant under a chiral transformation, making thus the chiral symmetry only an approximate rather than an exact symmetry.
2. Verify explicitly that in the perturbative $U(1)$ example of Sec. 6.2.1 the NBG disappears from the potential (but not from the kinetic term). Compute explicitly the mass of the h scalar.

3. Show that expanding the NGB exponential up to $\mathcal{O}(\pi^2)$, we have

$$\begin{aligned} d_\mu^\alpha &= \frac{\partial_\mu \pi^\alpha}{f} - \frac{f^{\rho\sigma\alpha}}{2f^2}\partial_\mu \pi^\rho \pi^\sigma + \dots \ , \\ E_\mu^A &= -\frac{f^{\alpha\beta A}}{2f^2}\partial_\mu \pi^\alpha \pi^\beta + \dots \end{aligned} \tag{3}$$

4. Using the chiral Lagrangian with M_q compute the meson masses in terms of the quark masses and of the parameters f and μ^2.

5. Show that the covariant derivative must take the form of Eq. (6.49) for $D_\mu U$ to transform as $D_\mu U \to g_L(x) D_\mu U g_R(x)$ with L_μ and R_μ transforming as normal gauge fields.

6. Consider an effective interaction between a neutral fermion χ and quarks of the form

$$\mathcal{L} = \frac{c}{\Lambda^2}\left(\bar{\chi}\gamma^\mu \chi\right)\left(\bar{q}\gamma^\mu q\right), \tag{4}$$

where q is the triplet of light quarks. The fermion χ has mass m_χ:

   - Compute the color and spin averaged cross section $q\bar{q} \to \chi\bar{\chi}$;
   - Write the chiral Lagrangian including such term. Which mesons decays are generated?
   - Although we have not discussed them in these lectures, vector mesons play an important role in low energy QCD. Compute the decay width of the $\rho \to \chi\bar{\chi}$ decay using the matrix elements listed in the Appendix of arXiv:2011.04735.

7 The Standard Model

1. Show explicitly that the values of the hypercharges listed in Eq. (7.68) are the correct ones for the quark fields.

2. Suppose that instead of taking $H \sim \mathbf{2}_{1/2}$ we allow for larger representations, $H \sim \mathbf{R}_{y_H}$. What is the value of the ρ parameter in this case? Show that only a handful of representations ensure $\rho = 1$ at tree-level.

3. Verify Eq.(7.85)

4. Check that baryon number and the individual lepton numbers are anomalous. What are the non-anomalous combinations that can be taken without considering gravitational anomalies? How does the inclusion of gravitational anomalies change the previous conclusion?

5. Show that the transformations of the fermions bilinear in Eq. (7.118) are correct.

6. Consider an extension of the SM in which a scalar particle $T \sim (1, \mathbf{3}, 0)$ is added.

   (a) Enumerate all new invariants that can be added to $\mathcal{L}_{SM}$;

   (b) What are the charges of the scalar fields contained in T? Compute them explicitly;

   (c) Compute the scalar mass matrix supposing $\langle T \rangle = 0$;

   (d) Compute the scalar mass matrix supposing $\langle T \rangle \neq 0$;

   (e) Is it true that in the last case the neutrinos get a non-vanishing mass? Justify your answer;

   (f) Compute $\Gamma(T^0 \to \nu\bar{\nu})$, where T^0 is the neutral component in T.

7. Suppose we extend the SM with an additional $U(1)_X$ symmetry, whose associated gauge boson will be called X_μ.

   (a) Is it true that we can write a term $B_{\mu\nu}X_{\mu\nu}$ in the Lagrangian? Why?

   (b) Is it true that we can write a term $W^A_{\mu\nu}X_{\mu\nu}$ in the Lagrangian? Why?

   (c) Suppose the Higgs boson is charged under the $U(1)_X$. What changes in the pattern of EWSB?

8. Compute the cross section $e^+e^- \to \mu^+\mu^-$ in the SM as a function of the center-of-mass energy $\sqrt{s}$.

9. Compute the annihilation cross section $e^+e^- \to W^+W^-$ in the SM as a function of the center-of-mass energy $\sqrt{s}$.

10. Compute the annihilation cross section $e^+e^- \to e^+e^-$ in the SM as a function of the center-of-mass energy $\sqrt{s}$.

11. We want to construct a theory based on the symmetry breaking pattern $SU(3)_1 \times SU(2)_2 \to SU(3)_c$, where $SU(3)_c$ is the usual color group. Which scalar representation would you add to the theory to achieve this symmetry breaking? Justify your answer.

12. How can we include the effects of weak interactions in the chiral Lagrangian?

8 Experimental confirmations of the Standard Model

1. Derive the decay width of the muon shown in Eq. (8.17).

2. Using the techniques we introduced in Chapter 6.3 compute the decay width $\Gamma(\pi^+ \to \pi^0 e\bar{\nu})$. *Hint:* it is convenient to introduce that coupling with the W boson using the same techniques used to include the coupling with photons.

3. Show that $\partial_\mu J_5^\mu \propto m_f$ where J_5 is the axial current.

4. Compute explicitly Eq. (8.39) and verify the this choice allows for the diagonalization of the kinetic Lagrangian.

5.
   - Compute explicitly the vacuum polarizations entering in the pole W mass;
   - Verify that all the divergences cancel out (pay particular attention to the role played by the CKM matrix).

6. Compute and check the formulas in Eq. (8.61).

9 Higher dimensional operators

1. Show that the Weinberg operator is the only dimension 5 operator that can be constructed with the SM field content and is invariant under $SU(3)_c \times SU(2)_L \times U(1)_Y$ transformations.

2. If neutrino masses are generated by the Weinberg operator the neutrinos are Majorana particles. How this changes the counting of the parameters appearing in the PMNS matrix as compared to the counting of parameters appearing in the CKM matrix?